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STATE OF THE ART OF THE THEORY AND ANALYTICAL DESIGN METHODS FOR VORTEX PUMPS

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STATE OF THE ART OF THE THEORY AND ANALYTICAL DESIGN METHODS FOR VORTEX PUMPS

ABSTRACT

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Recent analytical treatment, primarily by the authors, of vortex (turbine, regenerative, periphery) pumps and centrifugal-vortex pumps is reviewed, accounting in greater detail than formerly for such factors as head, capacity, and efficiency requirements, impeller blading, outer channel cross section configuration, leakage and other losses, recirculation, shaft r.p.m., etc. Analytical and experimental characteristic curves are given for specific Soviet models, with fair consistency between the two. High efficiencies are obtained using an open impeller with 24 back-curved blades and a rectangular channel (43%), or a closed impeller with semicircular passage and 24 to 32 forward-curved blades (50%). The theory presented herein is adequate for an analytical determination of the mean vortex velocity and ultimately for derivation of a fundamental equation of the vortex pump. author

Vortex and centrifugal-vortex pumps currently enjoy widespread application /20* in several branches of industry and agriculture. The first prototypes of vortex pumps were designed in 1930. For 27 years, however, the theoretical analysis of the operation of vortex pumps has lagged considerably behind their utilization, which has in turn held back the design of newer, more improved pump constructions.

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*Numbers in the margin indicate pagination in the original foreign text.

In the Soviet literature (refs. 1 to 7), these pumps are referred to as vaned or vortex pumps, in the German literature (refs. 8 and 9) as self-priming centrifugal pumps with peripheral channels, or Sihi pumps, in the American literature (refs. 10 and 11) as volute type vortex pumps, tangential pumps, vaned turbine pumps, regenerative, turbulent, or friction pumps, and in the Japanese literature (refs. 12 and 13) as rotary, friction, or periphery pumps. The diversity of nomenclature simply bears witness to the desire to distinguish these pumps from among other types, and the complexity of operation of vortex pumps.

The term "vortex pump" proposed by Prof. S. S. Rudnev is the most appropriate, since it most completely embraces the principle of operation. Vortex pumps are used predominantly in the realm of low specific speeds ($n_s = 4$ to 50), wherever the application of centrifugal pumps is rendered unsuitable due to low efficiency, fabrication complexity, and the lack of self-priming. Given the same impeller diameter and shaft r.p.m., the vortex pump generates a head three to five times as great as the centrifugal counterpart, hence it is normally designed to operate at high heads (25 to 250 m) and relatively low capacity (2 to 60 m^3/h).

At the present, three types of vortex pump are in use (fig. 1). The first includes pumps with a blind peripheral passage, where the suction and discharge openings are located at a lesser radius than the peripheral channel. The second and third types are pumps with a one- or two-directional open peripheral passage, where the suction and discharge openings are situated immediately at the beginning and end of the channel or the suction opening is located at a lesser radius. Pumps of the first type have a self-priming feature, while pumps of the second and third types are deprived of this feature (without auxiliary equipment).

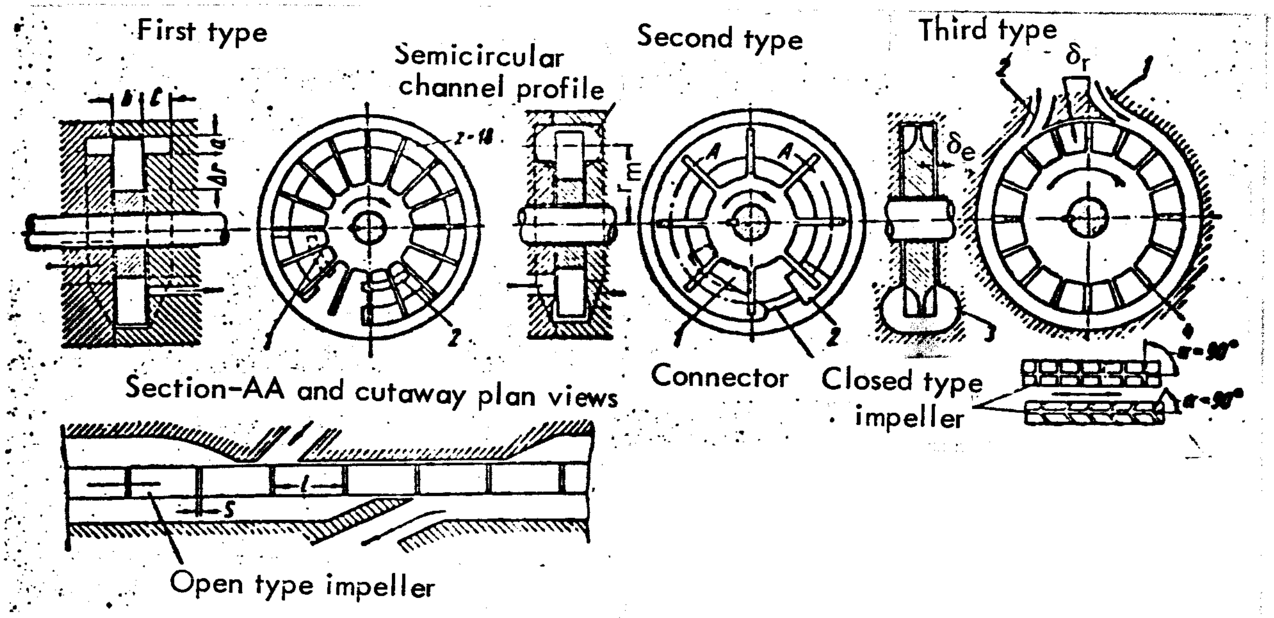

Figure 1. Diagrams of Vortex Pumps: 1) Suction; 2) Discharge; 3) Peripheral Channel; 4) Impeller; a = Channel Width; b = Impeller Width; c = Channel Depth; Δr = Distance from Impeller Hub to Channel; s = Blade Thickness; ι = Blade Pitch; δ_e = End Clearance; δ_r = Radial Clearance.

The present article gives a concise critical analysis of the basic hypotheses underlying the operating principle of vortex pumps, practical recommendations for the engineering calculation and design of three types of such pumps, and the scheme adopted by the authors for describing their operation.

The most detailed investigations of the working process of vortex pumps during the period 1939 to 1950 were undertaken by G. T. Berezyuk (ref. 1), O. V. Baybakov (ref. 2), and B. I. Nakhodkin (ref. 3).

In the opinion of G. T. Berezyuk, an increase in head in the peripheral channel of a vortex pump (first type) is realized as a result of the onset of secondary currents, as well as the entraining action of the impeller and strong turbulence. Limiting himself to these general considerations, G. T. Berezyuk gave no specific rationale for the working process that would be amenable to mathematical treatment. He was therefore unable to give the functional relationship between the pump head and those factors contributing to it.

O. V. Baykov investigated several vortex pumps of the first and second types and attempted to derive a fundamental equation for the vortex pump, i.e., the dependence of the head on the pump capacity and geometry of the flow-through portion.

Relying entirely on the hypothesis of Schmiedchen (ref. 9), O. V. Baykov /21 wrote the momentum imparted to the fluid by the impeller per second over the working length of a channel with a central angle ϕ:

$$M=\rho\int_{R_i}^{R_O}\int_{0}^{\varphi}v_u v_m d\varphi R dR,$$

where ρ is the density of the fluid, R_i and R_o are the inside and outside radii of the peripheral channel, v_u and v_m are the peripheral and meridional components of the velocity.

Substituting into this equation the values of M and o, we obtain the increment in head over the angular section ϕ_0:

$$\Delta h=\frac{1}{gF}\int_{R_i}^{R_O}\int_{0}^{\varphi}v_u v_m R d\varphi dR,$$

where g is the gravitational acceleration, F is the cross section of the peripheral channel.

This equation, which was integrated by O. V. Baybakov with a number of simplifications, cannot be used for practical purposes, since the fluid rotational velocity v_m in the plane of the channel cross section F remains unknown.

V. I. Nakhodkin systematized and generalized a wealth of experimental data on vortex pumps of the first type, experimentally investigated the balance of energy in a vortex pump, and conducted a series of tests to find out how the shape of the peripheral channel affected the performance curve of the vortex pump. B. I. Nakhodkin based his work on the assumption that the total head of the vortex pump is made up of the head H developed by the impeller in the suction and discharge section (as occurs in the conventional centrifugal pump) and the vortex head H_k obtained as the result of energy transfer from the rapidly moving fluid particles in the impeller vanes to the slowly moving liquid in the channel in connection with the intense formation and breakdown of vortices in the working section of the channel.

Forsaking any attempt at deriving the fundamental vortex pump equation analytically, B. I. Nakhodkin undertook a calculation of the performance curves for model pumps, applying similarity principles. In the event that no model was available, he recommended analyzing the vortex pump according to the following procedure.

1. The pump head is calculated as $H = k_0 u^2/2g$, where k_0 is the pressure coefficient as determined from an empirical-statistical graph of $k_0 = f_1(n_s)$.

2. The pump capacity is determined from the equation

$$Q_0 = \left(\frac{c}{u}\right)_0 uF,$$

where $(c/u)_0$ is the coefficient of discharge as determined from an empirical-statistical graph of $(c/u)_0 = f'_2(n_s)$, u is the mean peripheral velocity of the impeller blade, c is the velocity of the fluid in the channel.

3. The optimum values of the various coefficients governing the cross sectional dimensions of the impeller and channel (fig. 1) and the number and pitch of the blades are chosen from graphs of those coefficients as functions of n_3. For example, according to the date of B. I. Nakhodnik, the optimum

values of the coefficients b/c and l/c for which the pump will have the highest pressure coefficients and efficiency are near unity, while the coefficient a/c amounts to 2 or 3. The relations that he advises for calculating the head and the graphs for determining the optimum values of the coefficients b/c, l/c, a/c, and L/l as a function of the specific speed of the pump are in good agreement with the experimental data that he obtained for vortex pumps of the first type, but cannot be used for calculations on pumps of the second and third types, which are the ones most widely used in actual practice.

For model TsVS-53 vortex pumps (n = 2980 r.p.m., D_2 = 82 mm) of the second and third types with a nonsemicircular channel profile, V. G. Kovalenko (ref. 4) came up with the following values for the coefficients (fig. 2). The lower values of the indicated coefficients are given by the more sloping Q-H and Q-N curves, the upper values by the steeply dropping curves.

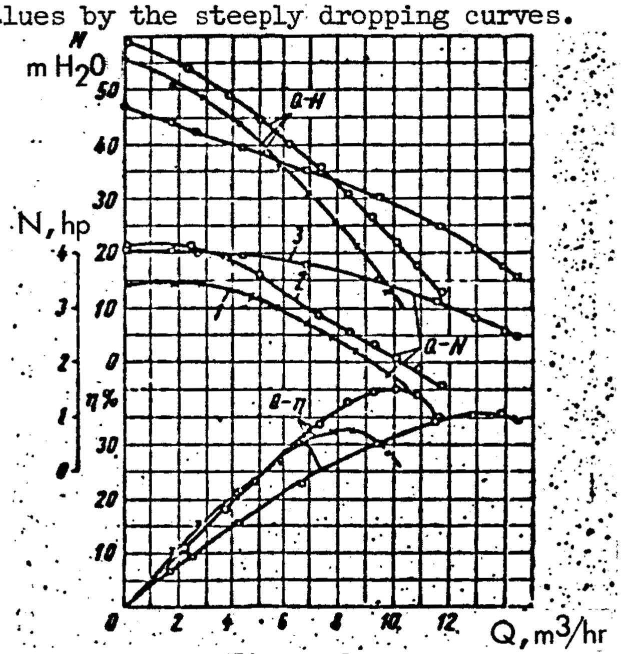

Figure 2.

Curve No.	$\frac{a}{c}$	$\frac{b}{c}$	$\frac{l}{c}$
1	3,0	1,2	1,5
2	3,5	1,2	1,6
3	2,0	0,85	1,1

The most noteworthy investigations of vortex pumps outside the Soviet Union in the last three or four years have been conducted by the American specialists Iversen (ref. 10) and Wilson, et al. (ref. 11). /22

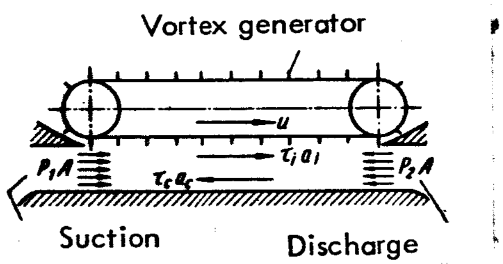

Figure 3.

Iversen, in explaining the operation of the vortex pump, advanced the hypothesis of the fluid mass becoming entrained in the channel due to shear stresses, which arise in the flow as the impeller interacts with the fluid, assuming that the impeller represents a kind of abstract rough surface. According to the diagram shown in figure 3, as the rough surface is set in motion, the balance of the forces acting on the fluid in the horizontal peripheral channel is expressed by the equation

$$P_1A - P_2A - \tau_c a_c + \tau_i a_i = 0,$$

where P_1 and P_2 are the pressures on the suction and discharge sides, respectively, A is the cross sectional area of flow τ_c is the shear stress on the channel wall of area a_c, τ_i is the shear stress on the impeller surface of area a_i.

The power delivered to the pumped fluid from the impeller surface moving with a velocity u is expressed by the equation

$$N = \tau_i a_i u.$$

For a fluid density ρ and mean fluid velocity v, the shear stress τ is arbitrarily determined by Iversen according to the equation

$$\tau = \frac{1}{2} c\rho v^2,$$

where c is an unknown coefficient.

Assuming then that, in determining the quantities τ_i and τ_c, the velocity v is the difference between the velocity of motion of the corresponding rough surface and the velocity of the fluid in the channel Q/A, Iversen obtains a fundamental vortex pump equation with two unknown coefficient c_i and c_c:

$$H = \frac{c_i a_i u^2}{2gA}\left[\left(1-\frac{Q}{uA}\right)^2 - \frac{c_c a_c}{c_i a_i}\left(\frac{Q}{uA}\right)^2\right].$$

Inasmuch as c_i and c_c are unknowns, the solution proposed by Iversen, like that of O. V. Baybakov, is not suitable for practical calculations, although it is based on a correct approach to analysis of the tangential stresses in the pump flow. Specifically, Iversen's error lies in his ignoring the causes giving rise to the tangential stresses (for example, the difference in the centrifugal forces at the impeller and in the channel), which was expressed in the representation of the impeller as an abstract rough surface.

Wilson investigated a number of industrial pump types and suggested that the fluid particles in the peripheral channel move in a helical trajectory. The helical motion of the fluid occurs at each point of the channel with a tangential velocity v_t and meridional velocity v_c directed perpendicular to v_t. Proceeding on this basis, Wilson introduces two concepts: 1) the tangential flow, defined as $Q = \int_{(A)} v_t dA$, where A is the cross section of the peripheral channel; this flow is assumed equal to the pump capacity; 2) the circulatory or meridional flow Q_c associated with the velocity v_c.

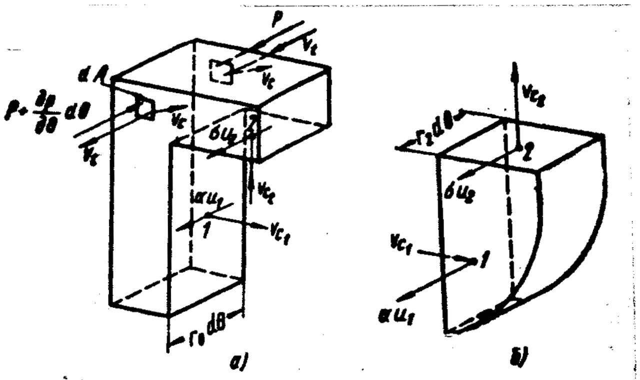

Figure 4

A theroetical investigation of the working process of the vortex pump was conducted by Wilson for a simplified model and for a linear law of fluid motion. Considering the elementary volume formed by a section of the peripheral channel (fig. 4a) and the working impeller (fig. 4b) of width $d\Theta$, and drawing on the equations of hydrodynamics, Wilson derived an equation for the head:

$$H = \frac{1}{g}\left[\frac{Q_c}{Q_s}\left(\sigma u_2^2 - \alpha u_1^2\right) - k_t\left(\frac{Q}{D^2}\right)^2\right],$$

where α and σ are coefficients denoting the ratio v_t/u at the points 1 and 2, respectively, Q_c is the circulatory capacity, which for the elementary volume is determined by the equation $dQ_c = v_{c2} r_2 d\Theta$, u_1 and u_2 are the peripheral velocities at the points 1 and 2, located at the radii r_1 and r_2, k_t is the tangential loss factor, D is the impeller diameter, Q is the pump capacity; $Q = \frac{1}{2} \cdot \frac{r_2}{r_G} Q_s \times$

$\times \left(k_1\sigma + \frac{r_1^2}{r_2^2} k_2\alpha\right)$ (here $Q_s = r_g A\omega$; r_G is the radius of the centroid of the area A, ω is the angular velocity of the impeller; k_1 and k_2 are dimensionless coefficients depending on the depth and width of the peripheral channel).

The following equation is given for determining the power: /23

$$N = \rho Q_s\left[gH + k_t\left(\frac{Q}{D^2}\right)^2\right].$$

The efficiency is calculated from the equation

$$\eta = \frac{Q}{Q_s} \cdot \frac{1}{\left(1 + \frac{k_t Q^2}{gHD^4}\right)}.$$

The fundamental equation expressing the relation between the head and discharge of fluid in the peripheral channel of the vortex pump is very complex and is not suitable for practical application, because, like the solutions of O. V. Baybakov and Iversen, it contains analytically indeterminate coefficients.

The theory and method of calculation proposed by Wilson have been subjected to rather severe criticism on the part of American specialists and have not been given a positive rating.

The foregoing brief analysis of the principal works in the theory and analytical methods for vortex pumps indicates that they are based on models of the vortex pump operation that are unsound for a variety of reasons and do not reflect completely the physical nature of the process involved. This renders them less valuable and makes their application for engineering calculations impossible. The experimental material acquired by various researchers is very meager and does not embrace all types of vortex pumps.

At the present time, there are no reliable data in the practice of vortex pump construction on the optimal relations of the impeller and channel dimensions for different types of vortex pumps. Data are lacking on the reconciliation of the dimensions and shape for which high efficiencies will be ensured at the highest heads and small size and weight of the pump. Finally, with the existing diversity of opinion regarding the nature of the vortex pump operation, there is no unified theory and method for their engineering calculations.

What theory and analytical method for the design of vortex pumps can be recommended for engineering purposes? Experimental design work and theoretical research carried out at the Krasnyy Fakel Factory on the working process of the vortex pump have fostered the following scheme to describe the operation of the vortex pump.

As the impeller moves, it creates vortices whose axes are parallel and perpendicular to the rotation axis of the impeller in the other two coordinate directions. Depending on the configuration and dimensions of the flow-through portion, particular vortices are prevelent, governing the dynamics of the working process and, consequently the analytical scheme.

The principal vortex formation that can be treated by any particular analytical scheme is accompanied by an additional vortex formation due to the impingement of fluid on the blade tips, roughness, etc., which cannot be accounted for analytically. In a properly designed manufactured pump, this additional vortex formation is a second-order quantity.[1] An increase in the mass of fluid pushed by the impeller is accompanied by partial or complete breakdown, deformation, or "detachment" of the vortices. If deformation of the vortices occurs, the consequence is that tangential stresses occur in the flow, causing the entrainment of fluid in the channel as the impeller moves. Since there are predominant vortices determing the qualitative and quantitative aspects of the working process, as well as secondary vortices which are unavoidable adjunct to the operation of any real pump, the tangential stresses must be arbitrarily broken down into principal and secondary components, the latter being analogous to the additional tangential stresses that arise in the turbulent motion of a fluid in pipes.

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[1]Such a pump must have a good streamline configuration in the flow-through part, minimum blade thicknesses and wall roughness, etc.

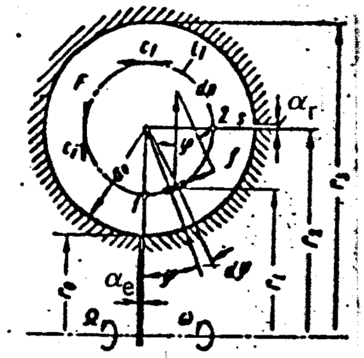

Figure 5.

In vortex pumps having a nearly semicircular channel cross section (fig. 5), especially favorable conditions are set up for vortices whose axis is directed perpendicular to the plane of this cross section. These vortices are caused by the action of varying centrifugal forces on the fluid mass in the vanes of the impeller and channel. The cross vortices with their axes in the other two perpendicular dimensions, given a sufficiently large number of thin blades, are negligible and will not be considered in the present article.

Pumps with an almost semicircular channel cross section have high heads and efficiencies, hence they are the most widely used in industry. The fluid pumped by them is of low viscosity, and the surfaces of the flow-through portion have little roughness. This justifies considering devoting some attention to the working process and analytical method for vortex pumps with a semicircular channel and flat radial blading working with an ideal fluid (ref. 6).

We will adopt the following notations for the elements of the peripheral channel and impeller (fig. 5):

f is the area of the impeller blade, s is the area of the blade tip in contact with the fluid, R is the radius of the center of gravity of the area F, r_c is the radius of the center of gravity of the area f, l_i is the projection of the streamline on the cross sectional plane of the channel, c_1 is the velocity of the fluid along l_i, α_r is the radial clearance between the casing and impeller, α_e is the end clearance between the casing and impeller, ρ is the density of the fluid, 1 is the point at which the fluid particles hit the blade, 2 is the point at which they depart from the blade.

We will assume for simplication that the fluid in the vanes of the impeller moves with a mean angular velocity ω, and that the mean angular velocity of the fluid in the peripheral channel is equal to (ref. 6)

$$\Omega = \frac{Q}{FR} .$$

In the absence of losses, $\Omega \to \omega$, and in the presence of resistance to the /24 motion, i.e., under any real working conditions, $\Omega < \omega$.

As already remarked, in the assumed scheme, the vortex motion in the pump channel is elicited by a difference in the centrifugal forces acting on the fluid masses in the channel and in the vanes of the impeller, with these masses moving at different angular velocities ω and Ω. Summing for the profile shown in figure 5 the elementary centrifugal forces dp sin ϕ acting along l_i over the entire cross section $(F + f)$, we obtain

$$P = \frac{2}{\pi}\rho f r_C(\omega^2 - \Omega^2). \qquad (2)$$

For profiles other than semicircular, in place of the factor $2/\pi$ we obtain, as the result of integration, the profile coefficient m, which can be used in equation (2) for convenience.

Hence, the centrifugal force P acting in the plane of the cross section $F + f$ gives rise to vortical motion with a certain mean velocity c_m. The mass per unit time M of the fluid taking part in this motion is defined as

$$M = \rho a c_m, \qquad (3)$$

where a is a length approximately equal to the profile radius a_0.

If the thrust P were not balanced during motion of the liquid by some resistive force Π, the motion would be expressed by the Newtonian equation

$$P = M\Delta c, \tag{4}$$

where Δc is the velocity increment that the mass M would acquire if it passed just once through the impeller.

This cannot occur, however, since the periodic acceleration of the liquid, which it can undergo as it repeatedly passes through the impeller in a vortex pump, would indicate a nonsteady state process, i.e.,

$$0 \neq \frac{\partial e}{He} \quad 0 \neq \frac{\partial e}{\partial e} \tag{5}$$

Consequently, the vortex motion must be described by the equation

$$P - \Pi = M\Delta c^*, \tag{6}$$

where

$$\Delta c^* = 0, \quad P = \Pi. \tag{7}$$

Formal application of the identity (6) will be of no avail in solving the problem. For this purpose, it is necessary to examine the relation of the vortex motion, as represented by the velocity c_m, and the peripheral motion, represented by the velocity v_u, which varies from the value v_{u2} at point 2 to v_{1u} at point 1 (fig. 6).

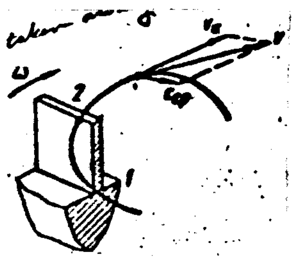

Figure 6.

How does the energy of the vortex motion acquired by the liquid directly from the impeller become transformed into the energy of peripheral motion, i.e., into the energy, part of which in the final analysis is imparted to the quantity of liquid Q in the form of head H? If the peripheral velocity of each fluid particle varies from v_{u2} to v_{u1} with a constant normal component of the velocity c_m, it then follows from the equation of motion that the flow has tangential forces

$$\tau = \rho c_m (v_{u2} - v_{u1}). \tag{8}$$

Integration of the tangential forces over the total area on which they act yields the mean tangential thrust

$$T = \rho c_m \left(u_c - \frac{Q}{F} \right) s \tag{9}$$

(per unit channel length $L \approx 2\pi R$).

Since the impeller is subjected to a thrust T with average velocity u_c, the power spent to accomplish this is

$$N = Tu_c. \tag{10}$$

According to the proposed scheme, the energy of peripheral motion is acquired through conversion of the corresponding amount of vortex energy. What amount of energy is sapped from the vortex motion? If we suppose (ignoring losses) that not all of the vortex energy is converted into peripheral kinetic energy, the surplus could be spent in accelerating the fluid mass M, i.e., in establishing $\Delta c \neq 0$. In this case the fluid velocity after passing through the impeller would be

$$c_1 = c_m + \Delta c, \tag{11}$$

where $\Delta c > 0$.

Considering that the motion would occur in this case under the action of a nonsteady state force P (since equilibrium is impossible according to the equation of motion (6)), the power of vortex motion would be

$$N_v = Pc_1 = P(c_m + \Delta c). \tag{12}$$

In reality, we have in the pump

$$\Delta c^* = 0 \tag{13}$$

and, hence,

$$N_{v*} = Pc_m. \tag{14}$$

Consequently, in order to ensure the conditions (13) and (14), the possible power difference

$$N_v - N_{v*} = P\Delta c \tag{15}$$

must be completely converted into the power of peripheral motion (10):

$$Tu_c = P\Delta c \tag{16}$$

or

$$Tu_c = \frac{P^2}{M}. \tag{17}$$

The additional equation (16), which is the result of applying the law of conservation of energy and the second law of Newton, makes it possible to determine the velocity of vortex motion c_m.

From equations (2) and (9) we find

$$c_m = \frac{mfu}{\sqrt{ss}}\left(1 + \frac{Q}{Fu_c}\right)\sqrt{1 - \frac{Q}{Fu_c}}. \tag{18}$$

Then, analyzing the equilibrium of the fluid volume in the channel under the action of the tangential moment $M = r_c T$ and the drag moment against the pressure of the pump $M_c = R\gamma HF$, and neglecting the moment due to friction of the fluid against the channel wall /25

$$r_c T = R\gamma HF, \tag{19}$$

we obtain the fundamental vortex pump equation

$$gH = ku_c^2(1 - \beta^2 x^2)\sqrt{1 - x}; \tag{20}$$

where

$$k = \frac{\psi L}{R}\frac{r_c}{R}\frac{mf}{F}\sqrt{\frac{s}{a}}$$

(ψ is the flow containment coefficient, L is the working length of the channel, m is the profile coefficient, which is calculated from the geometry);

$$\beta = \frac{r_g}{R};$$

$$x = \frac{Q}{Fu_c}$$

We obtain an expression for the expended power by multiplying the moment M_c by the angular velocity of the impeller:

$$N = \gamma HFR\omega. \tag{21}$$

The useful power of any pump, of course, is equal to

$$N_u = \gamma HQ. \tag{22}$$

Hence the efficiency of the working process of the vortex pump is

$$\eta_{pr} = Q/FR\omega. \tag{23}$$

The overall efficiency of the vortex pump should take into account the leakage, mechanical and hydraulic losses:

$$\eta = \eta_{pr}\,\eta_o\,\eta_{mech}\,\eta_{hyd}. \tag{24}$$

Equations (20), (21), and (23) give a unified solution to the vortex pump problem, i.e., they uniquely determine its head, power, and efficiency as functions of the capacity for given dimensions and pump r.p.m.

An analysis of the curves calculated according to equations (20) and (21) show that their behavior corresponds quite well to the behavior of the experimental curves shown in figures 2 and 7 to 10.

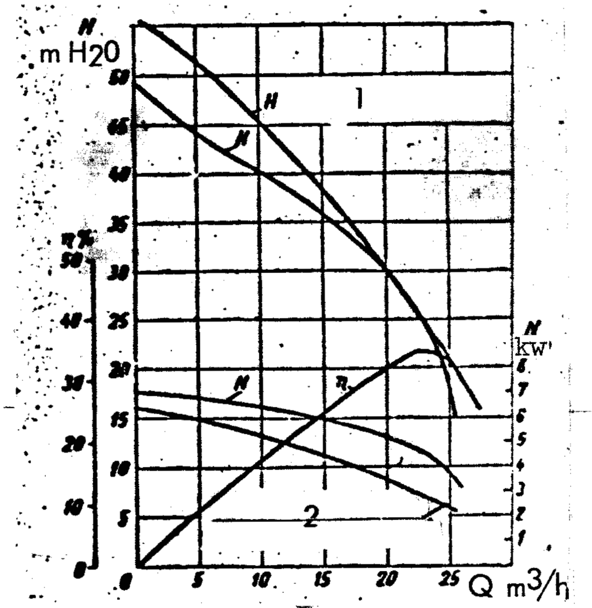

Figure 7. Characteristic Curves of the 2.5VS-3 Pump for Water at N = 1450 r.p.m. Without Aspiration.

1) H without leakage (calc.)

2) N without mechanical loss or leakage (calc.)

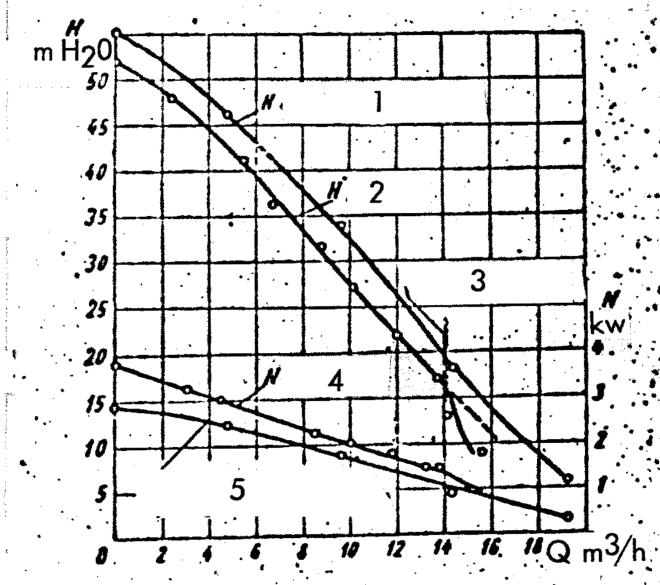

Figure 8. Experimental and Calculated Characteristics of the 1.5V-1.3M Pump at n = 1450 r.p.m.

1) H without leakage (calc.)

2) H (exp.)

3) Onset of cavitation, experimental pump

4) N (exp.)

5) N without mechanical loss or leagage (calc.)

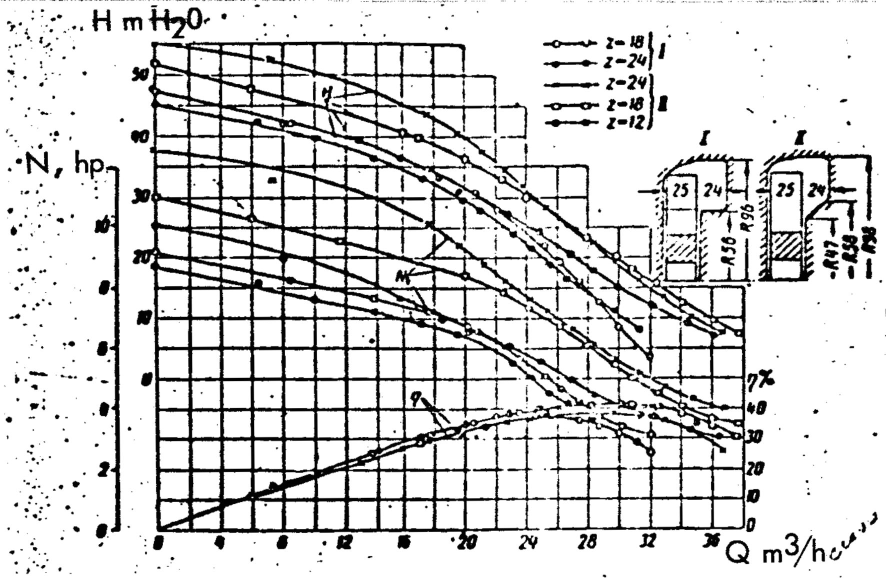

Figure 9. I) VS-65 Pump; II) VS-65AM Pump.

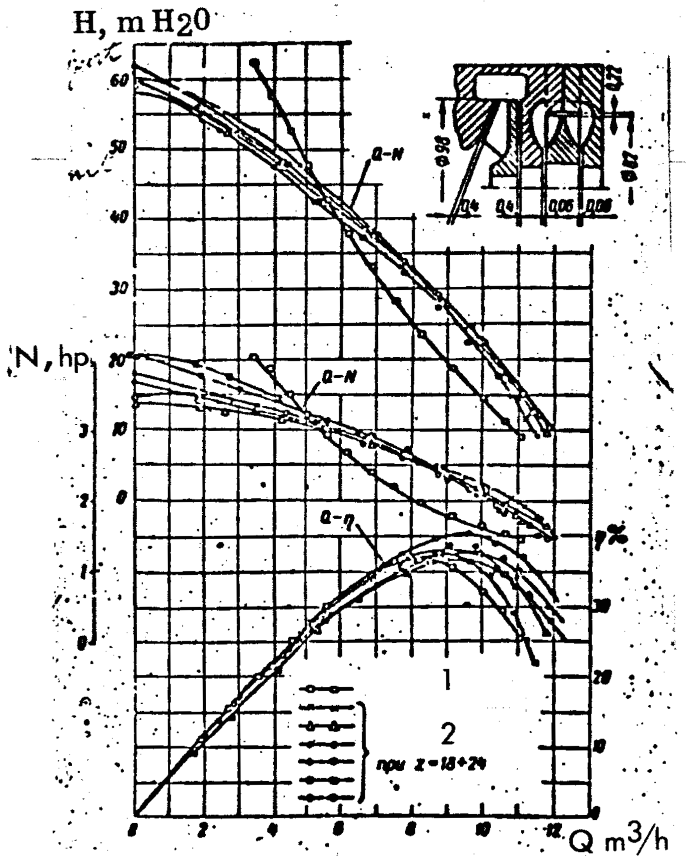

Figure 10. TsVS-53 Pump at n = 2980 r.p.m.;

1) "BACK ANGLE" 2) "FORWARD ANGLE" for z = 18 - 24

If the channel profile is not markedly different from semicircular, the checking and design calculations carried out for a number of Soviet vortex pump models at n = 1500 r.p.m. nearly coincide with the experimental data. In this case, the fundamental equation (20) can be used without any correction factors.[2] If the channel profile is appreciably nonsemicircular (see fig. 1), it is necessary to use the empirical coefficients given above.

The above solution of the vortex pump problem implies the following order-of-magnitude choice and calculation of a projected pump.

One must first decide on the type of pump, whether vortex, centrifugal or displacement. This choice is stipulated by many factors: dimensions, weight, attainable efficiency, self-priming requirements, useful life, viscosity of the fluid to be pumped, its temperature, contamination, length of operation, absolute head, capacity, etc.

The approximate limits of applicability of vortex pumps can be delineated here: fluid viscosities up to 5°E; coarseness of mechanical impurities up to 0.1 mm; head on a single impeller from 1 to 200 m; capacity from 0.1 to 17 liter/sec; peak efficiencies to 45%; minimum outlay of metal per kW power (minimum size and weight) down to 2 kg/kW; specific speed n_s from 4 to 50; shaft /26 r.p.m. to 6000.

In the event that vortex and centrifugal-vortex pumps are equal applicable to a specific situation, the choice is determined by two considerations: high suction and attainable efficiency. As is known, the centrifugal stage builds

[2] The disparity between the experimental and calculated curves in figure 7 is attributable to leakage in a real pump as $Q \to 0$ and to cavitation when $Q \to 25\ m^3/h$ in figure 7 and for $Q \to 14\ m^3/h$ in figure 8.

up the necessary head for the vortex stage and improves the overall efficiency of the system (ref. 5). The vortex stage in this case may be of prime or secondary importance.

Space limitations prevent us from examining all possible versions of pump construction. We will therefore confine our analysis to just those cases when the parameters of the vortex stage have already been chosen on the basis of adequate considerations and the problem is to determine the shape and dimensions of the flow-through section, then to derive the pump performance curve analytically.

The decisive factor in selecting the dimensions of the vortex stage is the attainment of high head. It is known that the head developed by a centrifugal pump is defined in the first approximation as

$$H \approx \frac{u_2^2}{2g}, \tag{25}$$

where

$$u_2 = \frac{\pi D_2 n}{60} \tag{26}$$

A calculation according to equation (20) with $Q = 0$, $\phi \to 1$, $= F$, $r_c = R$, $\beta = 1$, $s \approx 2a$ yields for the vortex pump.

$$2k_{e_{max}} \approx 18 \tag{27}$$

or

$$H_{e_{max}} \approx 18\frac{u_c^2}{2g} \tag{28}$$

where

$$u = \frac{2\pi r_c n}{60} \tag{29}$$

It is known that the capacity at η_{max} for a vortex pump corresponds largely to

$$Q_0 \approx 0.5 F u_c. \qquad (30)$$

Assuming for the first approximation that the Q-H curve is a straight line, we define

$$H_0 \approx 9 \frac{u_c^2}{2g}. \qquad (31)$$

Hence, by knowing H_0 and the r.p.m. n, we find r_c from equation (29). Then, from the prescribed capacity Q_0, knowing u_c, we determine F from equation (30). Since we have assumed $f = F$ (semicircular profile), we find

$$2f = \pi a_0^2 \qquad (32)$$

from which we ascertain the profile radius a_0.

This profile is basic to the subsequent pump design. It must be taken into account that the internal leakage via the clearances α_e and α_r, the containment of the flow by the blades, and the reduction in working length of the channel L due to the necessary linkage (connector) between the suction and discharge openings lower the head of the pump. Consequently, the impeller diameter must be increased accordingly. /27

For the corrected profile, the Q-H and Q-N curves can be constructed from equations (20) and (21) without regard for leakage or hydraulic and mechanical losses, the determination of which we will not consider in the present article.

If now some kind of special requirements are imposed on the pump characteristic curve in addition to the generation of high head and efficiency, its profile must depart considerably from semicircular. In this case, the expression

$$k = \frac{\psi L}{R} \cdot \frac{r_a}{R} \cdot \frac{mf}{F_1} \sqrt{\frac{s}{a}} \tag{33}$$

provides a sensible approach to the variation in dimensions and shape of the flow-through part of the pump in order to refine its characteristic.

This variation generally proceeds in two directions in vortex stages which function as the secondary stage (either it is required to obtain a maximally sloping Q-H characteristic or to minimize the power utilized by the pump). Such requirement are often imposed on self-priming vortex stages, where the efficiency and head are of secondary importance.

Equation (33) shows that by varying the ratio of the blade area f to the channel cross section F, the pump characteristic can be sharply altered. In the limit $F \to \infty$, $H \to 0$, $Q_{max} = Fu_c \to \infty$, i.e., the Q-H characteristic tends toward the horizontal axis.

It is also possible to affect the pump characteristic by varying the number of blades, their angle of inclination, the length of the channel, and the tips of the blades s, the dimensions and location of the suction and discharge points, the ratio r_c/R, and shape of the profile. The more the profile differs from semicircular, the greater will be the vortex drag. Inasmuch as the velocity c_m increases abruptly as $Q \to 0$ according to equation (17), the head and power input are again minimized.

In practice, the impellers of vortex pumps are normally found in two modifications, open and closed impellers. The open impeller represents a hub with long radial blades, which are bounded at the sides and periphery by the peripheral channel and flat walls of the pump casing. The closed impeller has a flat disk with short blades, situated at the periphery on both sides and separated at the middle by a connector. The angle α between the blade and connector varies within the limits 60 to 90°. These impellers are made with blades of various cross sectional shapes - rectangular, trapezoidal, crescent-shaped. The most popular are blades with a rectangular or trapezoidal cross section and curved backward. The number of blades on an open impeller is usually from 12 to 24, on a closed impeller it is from 18 to 38.

From the qualitative aspect, the number of blades affects the performance curve as follows. Increasing the number of blades within the indicated limits and preserving the geometry of the peripheral channel increases the head and capacity of the pump (see fig. 9), while the optimum value of the efficiency is displaced toward higher capacities; a decrease in the number of blades has the reverse effect.

The highest efficiencies (to 43%) are given by vortex pumps with an open impeller at $z = 24$ and with a rectangular channel cross section, i.e., with an optimum value of the coefficient $l/c = 1$. The blades in this case are usually curved back. However, with the proper combination of shape and geometric dimensions of the peripheral channel for the number of blades $z = 12$, vortex pump characteristics can be obtained identical to the characteristics of pumps with $z = 24$.

The difference between the characteristic curves of the indicated pumps is contained in the fact that a pump with $z = 24$ develops considerably larger heads at low capacities than the pump with $z = 12$, due to the "looser" connection between the suction and discharge sides.

The closed type of impeller combines well in the hydraulic sense with peripheral channels having a semicircular cross section or nearly so. A similar increase in the number of blades increases the head, capacity, and efficiency of the pump in this case, the highest efficiency (to 50%) resulting from an impeller with z = 24 to 32 at an angle $\alpha = 65$ to 70° and "forward" rotation of the impeller. With "backward" rotation, all other conditions being equal, for certain pump models the head and power rise sharply, while the efficiency is reduced 3 to 5% and the optimum value shifts toward lower capacities. In the first instance, the curvature of the Q-H and Q-N characteristics is up, in the second instance it is down, while in the interval of low capacities it asymptotically approaches the ordinate axis (see fig. 10).

This attribute of the impeller is very widely taken advantage of in industry, because a simple turning of the impeller through 180° on the pump shaft can result in high heads at low capacities, without any change whatsoever in the construction of the pump.

Consequently, the number of blades and their angle of inclination, like the geometry of the peripheral channel, exert a significant influence on the generation of good vortex pump characteristic curves.

It should be pointed out in conclusion that the theory presented herein to describe the operation of vortex pumps permits the velocity of the vortex motion c_m of the fluid in the pump to be determined analytically. The value found for the velocity c_m enables one to derive the fundamental equation of the vortex pump for construction of a theoretical characteristic curve $H = f(Q)$ on the basis of the prescribed dimensions.

A comparison of the calculated data based on equations (20) to (22) with the experimental data disclose satisfactory agreement for a series of various industrial models of vortex pumps. A qualitative estimate of the effect of the pump geometry on its characteristic curve according to equation (20) permits a rational approach to the design of new vortex pump constructions of low weight and size with enhanced hydraulic and operational characteristics.

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